Int J Theor Phys (2009) 48: 2473–2485
DOI 10.1007/s10773-008-9925-5

# Quantum Field Formalism for the Electromagnetic Interaction of Composite Particles in a Nonrelativistic Gauge Model III

**E.C. Manavella · R.R. Addad**

Received: 20 October 2008 / Accepted: 18 December 2008 / Published online: 8 May 2009


**Abstract** A classical nonrelativistic $U(1)\times U(1)$ gauge field model that describes the topologically massive electromagnetic interaction of composite particles in $(2+1)$ dimensions is proposed. The model, generalization of a previously postulated one, contains a Chern-Simons $U(1)$ field and the topologically massive electromagnetic $U(1)$ field, and it uses both a composite boson system or a composite fermion one. The second case is considered explicitly. By using the Dirac Hamiltonian method for constrained systems, the canonical quantization is carried out. By means of the Faddeev-Senjanovic formalism, the path integral quantization is developed. Consequently, the Feynman rules are established and the diagrammatic structure is treated. The application of the Becchi-Rouet-Stora-Tyutin algorithm is discussed. The present and previous models are compared.

**Keywords** Quantum field theory · Composite bosons and fermions

## 1 Introduction

As it is well-known, the composite bosons and fermions (CB, CF) theory [18, 19, 22–25, 29, 32, 35–37, 40–43, 46] has significant relevance and current importance in the understanding of the quantum Hall effect in its integer and fractional aspects.

We are interested in studying the electromagnetic interaction of composite particles in $(2+1)$ dimensions.

Therefore, we proposed [26] a classical nonrelativistic $U(1)\times U(1)$ gauge field model which contains two $U(1)$ gauge fields, a Chern-Simons (CS) field $a_\mu$ [2, 45, 47] and the electromagnetic field $A_\mu$.

E.C. Manavella (✉) · R.R. Addad
Facultad de Ciencias Exactas, Ingeniería y Agrimensura, Universidad Nacional de Rosario, Av. Pellegrini 250, 2000 Rosario, Argentina
e-mail: manavella@ifir-conicet.gov.ar

E.C. Manavella
Instituto de Física Rosario, Universidad Nacional de Rosario, Bv. 27 de Febrero 210 bis, 2000 Rosario, Argentina

Likewise, we performed the canonical quantization for the model. This was made by means of the Dirac Hamiltonian formalism for constrained systems [8, 9, 38].

Furthermore, we implemented the path integral quantization method through the Faddeev-Senjanovic (FS) procedure [10, 34] because the model has first and second-class constraints. Consequently, we established the Feynman rules of the model.

Moreover, we considered a simplified version of the model related to one used within the framework of condensed matter [17].

Also, we applied the Becchi-Rouet-Stora-Tyutin (BRST) algorithm to the model [4, 5, 15, 16, 20, 28, 38, 44].

We treated explicitly the CF case.

On the other hand, we proposed to extend the model by adding different terms to the Lagrangian density.

Thus, in [27], we considered a topological mass term for the electromagnetic field and interaction terms between the CS and electromagnetic fields.

Besides, we analyzed the obtained model following the same steps as the ones carried out in [26] and we compared the obtained and original models.

Furthermore, we found that the equations corresponding to the pure interactive case can be obtained directly from the ones corresponding to Manavella and Addad [27], by cancelling the terms with topological mass. On the contrary, the equations corresponding to the pure topologically massive case cannot be found from the ones corresponding to that reference by cancelling the interaction terms between the gauge fields.

For this reason, the purpose of the present paper is to consider the pure topologically massive case and to analyze the corresponding model in similar way to the above papers. Some results were given in advance in [27].

The paper is developed as follows: In Sect. 2, we describe classically our model and we make its canonical quantization through the Dirac algorithm. Next, in Sect. 3, by using the path integral quantization procedure by means of the FS formalism, we establish the Feynman rules of the model and we analyze its diagrammatic structure. Then, in Sect. 4, we apply the BRST method to the model. Afterwards, in Sect. 5, we compare the present and original models. Finally, in Sect. 6, we present our conclusions and outlook.

## 2 Classical Model and Canonical Quantization

As it is well-known, the addition of a CS term to the Maxwell action leads to the topologically massive $(2+1)$-dimensional electrodynamics [21]. In this theory, a modified Gauss law appears with the result that any charged particle carries a magnetic flux proportional to its charge.

In the present paper, as it was said in the introduction, we consider a topological mass term for the electromagnetic field. Consequently, we use the electrodynamics above mentioned.

Hence, we propose a classical nonrelativistic field model with $U(1) \times U(1)$ gauge symmetry for the topologically massive electromagnetic interaction of composite particles in $(2+1)$ dimensions. We explicitly consider a CF system. We think that this system can be treated through the following singular Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{cf}^{em} + \mathcal{L}_{tm}, \tag{2.1}$$

where $\mathcal{L}_{cf}^{em}$ is given by

$$\mathcal{L}_{cf}^{em} = i\psi^\dagger \mathcal{D}_0 \psi + \frac{1}{2m_b}\psi^\dagger \vec{\mathcal{D}}^2 \psi - \mu\psi^\dagger\psi + \frac{1}{4\pi\tilde{\phi}}\varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \tag{2.2a}$$

and $\mathcal{L}_{tm}$ is written as

$$\mathcal{L}_{tm} = \frac{1}{2\sigma}\varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{2.2b}$$

In (2.2), the Greek indices take the values $\mu, \nu, \rho = 0, 1, 2$.

We use natural units where $\hbar = c = 1$. The Minkowskian metric is $g_{\mu\nu} = \mathrm{diag}(1, -1, -1)$ and $\varepsilon^{012} = \varepsilon^{12} = 1$.

In (2.2a), the covariant derivative which involving both the CS $U(1)$ gauge field $a_\mu$ and the topologically massive electromagnetic $U(1)$ gauge field $A_\mu$ is given by $\mathcal{D}_\mu = \partial_\mu - i a_\mu - ieA_\mu$ (we take the electron charge as $-e$) and furthermore $\vec{\mathcal{D}}^2 = \mathcal{D}_1^2 + \mathcal{D}_2^2$. The matter field $\psi$ is a charged spinorial field describing CF. $m_b$ and $\mu$ are the band mass and the chemical potential of the electrons, respectively. $\tilde{\phi}$ is the strength of the flux tube in units of the flux quantum $2\pi$. (We fix the strength for the fictitious charge of each particle that interacts with the fictitious gauge field in the unit.)

In (2.2b), the first term on the right-hand side is the topological mass term for the electromagnetic field. The topological mass is given by $2\pi/\sigma$ and so the real magnetic flux bound to the electrons is $e\sigma/2\pi$. In the second term on the right-hand side of that equation, $F_{\mu\nu}$ is the electromagnetic field tensor.

Therefore, the Lagrangian density corresponding to the original model is obtained from (2.1) by removing the topological mass term.

By means of the expression for the covariant derivative, we rewrite (2.2a) as follows:

$$\begin{aligned}\mathcal{L}_{cf}^{em} = {} & i\,\frac{\tau+1}{2}\,\psi^\dagger\,\partial_0\psi + i\,\frac{\tau-1}{2}\,\partial_0\psi^\dagger\,\psi + \psi^\dagger\,(a_0 + eA_0)\,\psi + \frac{1}{2m_b}\,\psi^\dagger\,\vec{\mathcal{D}}^2\psi - \mu\,\psi^\dagger\,\psi \\ & + \frac{1}{4\pi\tilde{\phi}}\,\varepsilon^{\mu\nu\rho}\,a_\mu\,\partial_\nu a_\rho. \end{aligned} \tag{2.3}$$

In this equation, the kinetic fermionic term is written in the general form through the arbitrary parameter $\tau$ [38].

Now, we are going to develop, by means of the Dirac method, the canonical quantization for the model.

The momenta $P^{\mathcal{I}} = (p^\mu, P^\nu, \pi_\alpha^\dagger, \pi_\beta)$ canonically conjugate to the independent dynamical field variables $A_{\mathcal{I}} = (a_\mu, A_\nu, \psi_\alpha, \psi_\beta^\dagger)$, respectively, are defined as $P^{\mathcal{I}} = \delta\mathcal{L}/\delta\dot{A}_{\mathcal{I}}$. In these equations, the compound index $\mathcal{I}$ runs over the components of the different field variables and the new Greek indices take the values $\alpha, \beta = 1, 2$.

The momenta are given by

$$p^0 = 0, \tag{2.4a}$$

$$p^i = \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij} a_j, \tag{2.4b}$$

$$P^0 = 0, \tag{2.4c}$$

$$P^i = \frac{1}{2\sigma}\varepsilon^{ij} A_j + F^{i0}, \tag{2.4d}$$

$$\pi_\alpha^\dagger = -i\frac{\tau+1}{2}\psi_\alpha^\dagger, \tag{2.4e}$$

$$\pi_\alpha = i\frac{\tau-1}{2}\psi_\alpha, \tag{2.4f}$$

where the Latin indices take the values $i, j = 1, 2$.

The nonvanishing fundamental equal-time ($x^0 = y^0$) Bose-Fermi brackets [6, 7] are written as

$$\left[a_\mu(x), p^\nu(y)\right]_- = \delta_\mu^\nu \delta(\vec{x}-\vec{y}), \tag{2.5a}$$

$$\left[A_\mu(x), P^\nu(y)\right]_- = \delta_\mu^\nu \delta(\vec{x}-\vec{y}), \tag{2.5b}$$

$$\left[\psi_\alpha(x), \pi_\beta^\dagger(y)\right]_+ = -\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.5c}$$

$$\left[\psi_\alpha^\dagger(x), \pi_\beta(y)\right]_+ = -\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.5d}$$

where the notation $[.,.]_\mp$ indicates brackets between bosonic and fermionic Grassmann variables, respectively.

From (2.4), we obtain the following primary constraints:

$$\Phi_1^0 = p^0 \approx 0, \tag{2.6a}$$

$$\Phi_2^{0i} = p^i - \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_j \approx 0, \tag{2.6b}$$

$$\Phi_3^0 = P^0 \approx 0, \tag{2.6c}$$

$$\Omega_\alpha^\dagger = \pi_\alpha^\dagger + i\frac{\tau+1}{2}\psi_\alpha^\dagger \approx 0, \tag{2.6d}$$

$$\Omega_\alpha = \pi_\alpha - i\frac{\tau-1}{2}\psi_\alpha \approx 0. \tag{2.6e}$$

The canonical Hamiltonian density is defined as $\mathcal{H}_c = \dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger\pi - \mathcal{L}$ which, by means of (2.4), remains

$$\begin{aligned}\mathcal{H}_c = &-\frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_0\partial_i a_j + p^i\partial_i a_0 - \frac{1}{2\sigma}\varepsilon^{ij}A_0\partial_i A_j - \frac{1}{8\sigma^2}A_i A^i - \frac{1}{2}P_i P^i + P^i\partial_i A_0 \\ &- \frac{1}{2\sigma}\varepsilon^{ij}A_i P_j + \frac{1}{4}F_{ij}F^{ij} + \mu\psi^\dagger\psi - \psi^\dagger\,(a_0 + eA_0)\,\psi - \frac{1}{2m_b}\psi^\dagger\vec{\mathcal{D}}^2\psi.\end{aligned} \tag{2.7}$$

Besides, the primary Hamiltonian density is defined as

$$\mathcal{H}_p = \mathcal{H}_c + \lambda_1\Phi_1^0 + \lambda_{2i}\Phi_2^{0i} + \lambda_3\Phi_3^0 + \lambda_\alpha^\dagger\Omega_\alpha + \Omega_\alpha^\dagger\lambda_\alpha, \tag{2.8}$$

where $\lambda_1$, $\lambda_{2i}$ and $\lambda_3$ are bosonic Lagrange multipliers and $\lambda_\alpha^\dagger$ and $\lambda_\alpha$ are fermionic ones.

Next, by implementing the consistency condition on the primary constraints, we obtain the following secondary constraints:

$$\Phi_1^1 = \left[\Phi_1^0, H_p\right] = \partial_i p^i + \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j + \psi^\dagger\psi \approx 0, \tag{2.9a}$$

$$\Phi_3^1 = \left[\Phi_3^0, H_p\right] = \partial_i P^i + \frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j + e\psi^\dagger\psi \approx 0, \tag{2.9b}$$

where $H_p = \int d^2x\, \mathcal{H}_p$ is the primary Hamiltonian.

We find that (2.9a) and (2.9b) are the time components of the equations of motion corresponding to $a_\mu$ and $A_\mu$, respectively.

When we impose the consistency on the constraints (2.6b), (2.6d) and (2.6e), the Lagrange multipliers $\lambda_{2i}, \lambda_\alpha^\dagger$ and $\lambda_\alpha$, appearing in (2.8), remain determined, respectively. Moreover, when we implement the consistency on the constraints (2.9), the resultant equations are satisfied automatically. Hence, there are no further constraints.

It is easy to show that the constraints (2.6a) and (2.6c) are first-class whereas the constraints (2.6b), (2.6d), (2.6e) and (2.9) are second-class. Nevertheless, the latter do not form a minimal set of second-class constraints. The reason is that the determinant of the matrix whose elements are the Bose-Fermi brackets between these constraints vanishes.

Therefore, there must be at least two linear combinations of second-class constraints which are independent of the above first-class constraints and these must also be first-class. We find that there are only two of such combinations, which are the following:

$$\Sigma_1 = e\Phi_1^1 - \Phi_3^1 = \frac{e}{4\pi\tilde{\phi}}\varepsilon^{ij}\partial_i a_j - \frac{1}{2\sigma}\varepsilon^{ij}\partial_i A_j + e\partial_i p^i - \partial_i P^i \approx 0, \tag{2.10a}$$

$$\Sigma_2 = \psi^\dagger\Omega - \psi\Omega^\dagger - \frac{i}{e}\Phi_3^1 = \psi^\dagger\pi - \psi\pi^\dagger - \frac{i}{e}\left(\frac{1}{2\sigma}\varepsilon^{ij}\,\partial_i A_j + \partial_i P^i\right) \approx 0. \tag{2.10b}$$

Consequently, two second-class constraints can be eliminated and so the final set of constraints remains:

(i) The first-class constraints defined by the functions $\Sigma_1$, $\Sigma_2$, $\Sigma_3 = \Phi_1^0$ and $\Sigma_4 = \Phi_3^0$. As it is well-known, these constraints are related to the symmetries of the gauge group $U(1)\times U(1)$ of the model.

(ii) The second-class constraints defined by $\Phi_2^{0i}$, $\Omega_\alpha^\dagger$ and $\Omega_\alpha$.

The extended Hamiltonian is defined by

$$H_e = \int d^2x\,(\mathcal{H}_c + \rho^a\Sigma_a) - \int d^2x\, d^2y \Gamma_I(x) F_{IJ}^{-1}(\vec{x},\vec{y})[\Gamma_J(y), H_c], \tag{2.11}$$

where $\rho^a, a = 1,\ldots,4$, are bosonic Lagrange multipliers, $F^{-1}$ is the inverse of the matrix $F$ whose elements are $[\Gamma_I, \Gamma_J]$, $I, J = 1,\ldots,6$, and $\Gamma_1 = \Phi_2^{01}$, $\Gamma_2 = \Phi_2^{02}$, $\Gamma_3 = \Omega_1^\dagger$, $\Gamma_4 = \Omega_2^\dagger$, $\Gamma_5 = \Omega_1$ and $\Gamma_6 = \Omega_2$ are the second-class constraints.

The matrix $F$ reads

$$F = \begin{pmatrix} 0 & -\frac{1}{2\pi\tilde{\phi}} & 0 & 0 & 0 & 0 \\ \frac{1}{2\pi\tilde{\phi}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 \end{pmatrix} \delta(\vec{x}-\vec{y}), \tag{2.12}$$

its determinant is

$$\det F = \frac{1}{4\pi^2\tilde{\phi}^2}\delta(\vec{x}-\vec{y}), \tag{2.13}$$

and its inverse is given by

$$F^{-1} = \begin{pmatrix} 0 & 2\pi\tilde{\phi} & 0 & 0 & 0 & 0 \\ -2\pi\tilde{\phi} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 \end{pmatrix} \delta(\vec{x}-\vec{y}). \tag{2.14}$$

Now, we calculate the Dirac brackets $D(F)$ with regard to the matrix $F$.

The $D(F)$ bracket between the functions $R(x)$ and $S(y)$ is defined as follows:

$$\left[R(x), S(y)\right]^{D(F)} = \left[R(x), S(y)\right] - \int d^2u d^2v \, [R(x), \Gamma_I(u)] \, F_{IJ}^{-1}(\vec{u}, \vec{v}) \left[\Gamma_J(v), S(y)\right]. \tag{2.15}$$

From this equation, we obtain the following nonvanishing $D(F)$ brackets:
field-field:

$$\left[a_1(x), a_2(y)\right]_-^{D(F)} = 2\pi\tilde{\phi}\delta(\vec{x}-\vec{y}), \tag{2.16a}$$

$$\left[\psi_\alpha^\dagger(x), \psi_\beta(y)\right]_+^{D(F)} = -i\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{2.16b}$$

field-momentum:

$$\left[a_0(x), p^0(y)\right]_-^{D(F)} = \delta(\vec{x}-\vec{y}), \tag{2.16c}$$

$$\left[A_\mu(x), P^\nu(y)\right]_-^{D(F)} = \delta_\mu^\nu\delta(\vec{x}-\vec{y}). \tag{2.16d}$$

When we impose the $D(F)$ brackets, we must take the second-class constraints as equations strongly equal to zero. Thus, the following field variables remain determined:

$$p^i = \frac{1}{4\pi\tilde{\phi}}\varepsilon^{ij}a_j, \tag{2.17a}$$

$$\pi_\alpha^\dagger = -i\frac{\tau+1}{2}\psi_\alpha^\dagger, \tag{2.17b}$$

$$\pi_\alpha = i\frac{\tau-1}{2}\psi_\alpha. \tag{2.17c}$$

Furthermore, the last term on the right-hand side of (2.11) vanishes and so the extended Hamiltonian remains

$$H_e = \int d^2x(\mathcal{H}_c + \rho^a\Sigma_a). \tag{2.18}$$

Now, we calculate the final Dirac brackets or simply, Dirac brackets.

The Dirac bracket between the functions $R(x)$ and $S(y)$ is defined as follows:

$$\begin{aligned} \left[R(x), S(y)\right]^{D} &= \left[R(x), S(y)\right]^{D(F)} \\ &\quad - \int d^2u\, d^2v\, [R(x), \Delta_A(u)]^{D(F)}\, G_{AB}^{-1}(\vec{u}, \vec{v}) \left[\Delta_B(v), S(y)\right]^{D(F)} \end{aligned} \tag{2.19}$$

In this equation, $G^{-1}$ is the inverse of the matrix $G$ whose elements are $[\Delta_A, \Delta_B]$, $A, B = 1, \ldots, 8$, $\Delta_a = \Sigma_a$ and $\Delta_{4+a} = \Theta_a$, where $\Theta_a \approx 0$ are admissible gauge-fixing conditions, one for each first-class constraint. The gauge-fixing conditions must verify that $\det G \not\approx 0$ and must be compatible with the equations of motion.

We choose the following gauge-fixing conditions:

$$\Theta_1 = \partial^i a_i \approx 0, \tag{2.20a}$$

$$\Theta_2 = \partial^i A_i \approx 0, \tag{2.20b}$$

$$\Theta_3 = a_0 \approx 0, \tag{2.20c}$$

$$\Theta_4 = \nabla^2 A_0 - \partial_i P^i + \frac{1}{2\sigma}\varepsilon^{ij}\,\partial_i A_j \approx 0, \tag{2.20d}$$

which satisfy the above requirements.

We find that the matrix $G$ is given by

$$G = \begin{pmatrix} 0 & 0 & 0 & 0 & e\nabla^2 & -\nabla^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{e}\nabla^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\nabla^2 \\ -e\nabla^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \nabla^2 & \frac{i}{e}\nabla^2 & 0 & 0 & 0 & 0 & 0 & \nabla^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \nabla^2 & 0 & -\nabla^2 & 0 & 0 \end{pmatrix} \delta(\vec{x} - \vec{y}), \tag{2.21}$$

its determinant is

$$\det G = -\left(\nabla^2\right)^6 \delta(\vec{x} - \vec{y}) \not\approx 0, \tag{2.22}$$

and its inverse reads

$$G^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & -e^{-1}u & 0 & 0 & 0 \\ 0 & 0 & 0 & -ieu & -iu & -ieu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \\ 0 & ieu & 0 & 0 & 0 & 0 & 0 & u \\ e^{-1}u & iu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ieu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.23}$$

where $u = -(2\pi)^{-1} \ln|\vec{x} - \vec{y}|$ and $v = \delta(\vec{x} - \vec{y})$.

Therefore, from (2.19) we find the following nonvanishing Dirac brackets:
field-field:

$$\left[a_1(x), a_2(y)\right]_-^D = 2\pi\tilde{\phi}\delta(\vec{x} - \vec{y}), \tag{2.24a}$$

$$\left[\psi_\alpha^\dagger(x), \psi_\beta(y)\right]_+^D = -i\delta_{\alpha\beta}\delta(\vec{x} - \vec{y}), \tag{2.24b}$$

field-momentum:

$$\left[A_i(x), P^j(y)\right]_-^D = \delta_i^j \delta(\vec{x} - \vec{y}) + \frac{1}{2\pi} \partial_i^x \partial^{xj} \ln|\vec{x} - \vec{y}|, \tag{2.24c}$$

momentum-momentum:

$$\left[P^1(x), P^2(y)\right]_-^D = -\frac{1}{2\sigma} \delta(\vec{x} - \vec{y}). \tag{2.24d}$$

When we impose the Dirac brackets, we must take the first-class constraints and the gauge-fixing conditions as equations strongly equal to zero. Hence, the following field variables remain determined:

$$p^0 = 0, \tag{2.25a}$$

$$P^0 = 0, \tag{2.25b}$$

$$a_0 = 0, \tag{2.25c}$$

$$A_0(x) = \frac{1}{2\pi} \int d^2y \left(-\partial_i P^i + \frac{1}{2\sigma} \varepsilon^{ij} \partial_i A_j\right)(y) \ln|\vec{x} - \vec{y}|. \tag{2.25d}$$

Furthermore, the second term on the right-hand side of (2.18) vanishes and so the extended Hamiltonian coincides with the canonical one.

Thus, the dynamics of the classical model remains completely specified.

Finally, the canonical quantization is made by replacing the Dirac brackets between field variables by the equal-time commutators or anticommutators between field operators according to the usual rule [9].

On the other hand, let us note that a CB system can be studied in the same way. In this case, the matter field is a charged scalar field. Therefore, the fermionic second-class constraints (2.6d) and (2.6e) turn into bosonic second-class ones. Besides, the fermionic brackets (2.5c), (2.5d), (2.16b) and (2.24b) become into bosonic ones.

## 3 Path Integral Quantization, Feynman Rules and Diagrammatic Structure

We use the Feynman path integral quantization formalism according to the FS method because the model has first and second-class constraints. For this reason, we write the generating functional of the model as follows:

$$\begin{aligned} Z = \int & \mathbb{D}a_\mu \mathbb{D}p^\mu \mathbb{D}A_\nu \mathbb{D}P^\nu \mathbb{D}\psi_\alpha \mathbb{D}\pi_\alpha^\dagger \mathbb{D}\psi_\beta^\dagger \mathbb{D}\pi_\beta \delta(\Gamma_I)(\det F)^{1/2} \delta(\Delta_A)(\det G)^{1/2} \\ & \times \exp\left[i \int d^3x \left(\dot{a}_\mu p^\mu + \dot{A}_\mu P^\mu + \dot{\psi}\pi^\dagger + \dot{\psi}^\dagger \pi - \mathcal{H}_e\right)\right], \end{aligned} \tag{3.1}$$

where $\delta(\Gamma_I)$ and $\delta(\Delta_A)$ stand for products of Dirac delta functions and the Hamiltonian density $\mathcal{H}_e$ was given in (2.11).

We can write $\delta(\Delta_8) = \delta(A_0 - f)$, where

$$f(x) = \frac{1}{2\pi} \int d^2y \left(-\partial_i P^i + \frac{1}{2\sigma} \varepsilon^{ij} \partial_i A_j\right)(y) \ln|\vec{x} - \vec{y}|. \tag{3.2}$$

Hence, by means of the Dirac delta $\delta(\Delta_8)$, we calculate the path integral over $A_0$.

Next, by proceeding analogously to [26], we find that the generating functional remains

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \delta(\partial^l a_l)\delta(\partial^m A_m)\exp\left(i\int d^3x \mathcal{L}\right), \tag{3.3}$$

where $\mathcal{L}$ is the starting Lagrangian density given by (2.1).

This result was expected. However, we considered necessary to start formally from the canonical path integral (3.1) and to prove that it is possible to arrive at the Lagrangian path integral (3.3). The reason is that, as it is well-known, there are many field theories in which the simple Lagrangian path integral cannot be obtained from the canonical one (see, for instance, [33] and references therein).

Finally, by using the Faddeev-Popov trick as in [26], the generating functional reads

$$Z = \int \mathbb{D}a_\mu \mathbb{D}A_\nu \mathbb{D}\psi_\alpha \mathbb{D}\psi_\beta^\dagger \exp\left(i\int d^3x \mathcal{L}_{eff}\right), \tag{3.4}$$

where the Lagrangian density $\mathcal{L}_{eff}$ is written in terms of the independent dynamical field variables, $a_\mu, A_\nu, \psi_\alpha$ and $\psi_\beta^\dagger$, and so it constitutes the effective Lagrangian density of the model. This is expressed as

$$\mathcal{L}_{eff} = \mathcal{L} + \mathcal{L}_{gf}, \tag{3.5}$$

where

$$\mathcal{L}_{gf} = \frac{\lambda_a}{2}\left(\partial^\mu a_\mu\right)^2 + \frac{\lambda_A}{2}\left(\partial^\mu A_\mu\right)^2 \tag{3.6}$$

is the gauge-fixing Lagrangian density.

Now, from (3.5), we calculate, similarly to [26], the propagators and vertices, in the momentum space, of the model.

As a result, the propagators $d_{\mu\nu}$ and $D_{\mu\nu}$ of the gauge fields $a_\mu$ and $A_\mu$, respectively, read

$$d_{\mu\nu}(k) = \frac{1}{\lambda_a}\frac{k_\mu k_\nu}{k^4} + 2i\pi\tilde{\phi}\varepsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}, \tag{3.7a}$$

$$D_{\mu\nu}(k) = g_{\mu\nu}\frac{1}{\frac{1}{\sigma^2} - k^2} + \left(\frac{1}{\lambda_A} - \frac{k^2}{\frac{1}{\sigma^2} - k^2}\right)\frac{k_\mu k_\nu}{k^4} + i\varepsilon_{\mu\nu\rho}\frac{k^\rho}{\sigma(\frac{1}{\sigma^2} - k^2)k^2}, \tag{3.7b}$$

where $k^2 = k_\mu k^\mu$.

The propagator $G$ of the matter field $\psi$ is written as

$$G(\vec{p}, E) = \left(E - \mu - \frac{\vec{p}^{\,2}}{2m_b}\right)^{-1}, \tag{3.8}$$

where $E$ is the particle energy, $\vec{p}$ its ordinary momentum and $\vec{p}^{\,2} = p_1^2 + p_2^2$.

The vectors $V^n = (V_\mu^n), n = 1, 2$, which represent the 3-point vertices of the model, read

$$V^1 = V, \tag{3.9a}$$

$$V^2 = eV, \tag{3.9b}$$

where

$$V = \left(1, \frac{1}{m_b} q_i\right), \tag{3.10}$$

with $i = 1, 2$.

Finally, the matrices $W_m = (W_m^{\mu\nu})$, $m = 1, 2, 3$, which represent the 4-point vertices, are written as

$$W_1 = -\frac{1}{2m_b} W, \tag{3.11a}$$

$$W_2 = -\frac{e^2}{2m_b} W, \tag{3.11b}$$

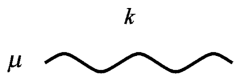
$$W_3 = -\frac{e}{m_b} W, \tag{3.11c}$$

where

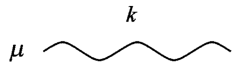
$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.12}$$

Next, we establish the Feynman rules of the model [39]:

(i) *Propagators.* We represent the propagators of the gauge fields $a_\mu$ and $A_\mu$ with a thick wavy line and a thin wavy line

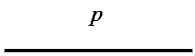


$\equiv d_{\mu\nu}(k)$,



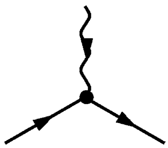
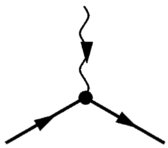
$\equiv D_{\mu\nu}(k)$,

respectively, and the propagator of the matter field $\psi$ with a straight line

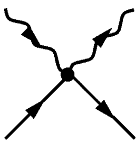
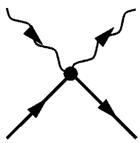


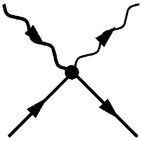
$\equiv G(\vec{p}, E)$.

(ii) *Vertices.* Thus, the 3-point vertices of the model remain represented by


$\equiv V_\mu^1$,

$\equiv V_\mu^2$

and the 4-point ones by


$\equiv W_1^{\mu\nu}$,

$\equiv W_2^{\mu\nu}$,



$$\equiv W_3^{\mu\nu}.$$

The remaining Feynman rules are the usual ones.

Next, in the framework of the perturbative theory, by means of a power-counting analysis, we find that the model has thirty-seven primitively divergent diagrams, twenty of them with two vertices and the rest with three.

In a future paper, we will develop the regularization and the renormalization of the model. Nevertheless, it is easy to show that this only has a finite number of divergent diagrams, i.e., it is a superrenormalizable model.

## 4 BRST Formalism

We note that the expressions obtained in Sect. 2 for $\mathcal{H}_c, \mathcal{H}_e, \Sigma_1, \Sigma_2$ and $\Theta_4$ are different from the ones corresponding to the original model [26]. Nevertheless, as it can be shown, for the BRST formalism, the mathematical developments used and the results found are exactly the same as the ones corresponding to such model.

## 5 Comparison between the Present and Original Models

Now, we are going to compare the found results with the ones corresponding to the original model [26].

In the present model, we have considered a topological mass term for the electromagnetic field, given by the first term on the right-hand side of (2.2b). Consequently, we have assumed that the real magnetic flux is attached to the particles. As we have seen in Sect. 2, by removing that term, we get the Lagrangian density corresponding to the original model.

Furthermore, in the present model, in contrast to the original one, the equations (2.23) and (2.25d), and consequently (2.24c) and (3.2), were obtained by considering rigorously the $(2+1)$-dimensionality of the model.

We find that the mathematical developments used and the results obtained for the present model extend the ones corresponding to the original model. In particular, the equations for the original model can be found directly from the ones corresponding to the present model by cancelling the terms with topological mass.

On the other hand, we find that the propagator of the electromagnetic field $D_{\mu\nu}$, given by (3.7b), has the same ultraviolet behavior that the one corresponding to the original model.

## 6 Conclusions and Outlook

By generalizing the model proposed in [26], a classical nonrelativistic $U(1) \times U(1)$ gauge field model that describes the topologically massive electromagnetic interaction of composite particles in (2+1) dimensions has been postulated.

Next, by following the Dirac method, the canonical quantization was performed. The model has ten constraints, four of them first-class and the rest second-class.

Afterwards, by means of the FS formalism, the path integral quantization procedure was developed. As a result, the Feynman rules of the model were established and its diagrammatic structure was analyzed. The model has five vertices, two of them with three points and the rest with four. Furthermore, the model has thirty-seven primitively divergent diagrams, twenty of them with two vertices and the rest with three.

Then, the application of the BRST algorithm to the model was briefly described.

Finally, the present and original models were compared.

The paper was developed considering explicitly the CF case.

In future papers, we will compare the obtained results with the ones corresponding to other models.

Besides, we will apply the found results within the framework of condensed matter.

Moreover, as we said in Sect. 3, we will analyze the perturbative development for the model.

On the other hand, as it is known, by adding higher-derivative terms for the gauge fields to the Lagrangian density of a model, keeping its gauge invariance, the ultraviolet behavior of the propagators corresponding to such fields can be improved. Consequently, the divergence of those diagrams in which these propagators take place can be eliminated [1, 3, 11–14, 30, 31]. For this reason, we think it is interesting to apply, in a future paper, this method to the model.

## References


1. Alvarez-Gaumé, L., Labastida, J.M.F., Ramallo, A.V.: Nucl. Phys. B **334**, 103 (1990)
2. Arovas, D.P., Schrieffer, J.R., Wilczek, F., Zee, A.: Nucl. Phys. B **251**, 117 (1985)
3. Avdeev, L., Grigoryev, G., Kazakov, D.: Nucl. Phys. B **382**, 561 (1992)
4. Becchi, C., Rouet, A., Stora, R.: Phys. Lett. B **52**, 344 (1974)
5. Becchi, C., Rouet, A., Stora, R.: Ann. Phys. (N.Y.) **98**, 287 (1976)
6. Casalbuoni, R.: Nuovo Cim. A **33**, 115 (1976)
7. Casalbuoni, R.: Nuovo Cim. A **33**, 389 (1976)
8. Dirac, P.A.M.: Can. J. Math. **2**, 129 (1950)
9. Dirac, P.A.M.: Lectures on Quantum Mechanics. Yeshiva University Press, New York (1964)
10. Faddeev, L.D.: Theor. Math. Phys. **1**, 1 (1970)
11. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Int. J. Theor. Phys. **34**, 1 (1995)
12. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Int. J. Theor. Phys. **34**, 1037 (1995)
13. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: J. Math. Phys. **37**, 84 (1996)
14. Foussats, A., Manavella, E., Repetto, C., Zandron, O.P., Zandron, O.S.: Specul. Sci. Technol. **20**, 3 (1997)
15. Fradkin, E.S., Fradkina, T.E.: Phys. Lett. B **72**, 343 (1978)
16. Fradkin, E.S., Vilkovisky, G.A.: Phys. Lett. B **55**, 224 (1975)
17. Halperin, B.I., Lee, P.A., Read, N.: Phys. Rev. B **47**, 7312 (1993)
18. Hansson, T.H., Chang, C.-C., Jain, J.K., Viefers, S.: Phys. Rev. B **76**, 075347 (2007)
19. Hansson, T.H., Chang, C.-C., Jain, J.K., Viefers, S.: Phys. Rev. Lett. **98**, 076801 (2007)
20. Henneaux, M.: Phys. Rep. **126**, 1 (1985)
21. Jackiw, R., Templeton, S.: Phys. Rev. D **23**, 2291 (1981)
22. Jain, J.K.: Composite Fermions. Cambridge University Press, Cambridge (2007)
23. Jeon, G.S., Chang, C.-C., Jain, J.K.: Eur. Phys. J. B **55**, 271 (2007)
24. Jiang, L., Ye, J.: Phys. Rev. B **74**, 245311 (2006)
25. Jolad, S., Chang, C.-C., Jain, J.K.: Phys. Rev. B **75**, 165306 (2007)
26. Manavella, E.C.: Int. J. Theor. Phys. **40**, 1453 (2001)
27. Manavella, E.C., Addad, R.R.: Int. J. Theor. Phys. **46**, 2868 (2007)
28. Marnelius, R.: Introduction to the quantization of general gauge theories. Preprint, Institute of Theoretical Physics, Göteborg (1981)
29. Milovanović, M.V., Stanić, I.: Phys. Rev. B **72**, 155306 (2005)

30. Nesterenko, V.V.: J. Phys. A: Math. Gen. **22**, 1673 (1989)
31. Odintsov, S.D.: Z. Phys. C **54**, 531 (1992)
32. Papić, Z., Milovanović, M.V.: Phys. Rev. B **75**, 195304 (2007)
33. Ryder, L.H.: Quantum Field Theory. Cambridge University Press, Cambridge (1996)
34. Senjanovic, P.: Ann. Phys. (N.Y.) **100**, 227 (1976)
35. Shi, C., Jeon, G.S., Jain, J.K.: Phys. Rev. B **75**, 165302 (2007)
36. Shi, C., Jolad, S., Regnault, N., Jain, J.K.: Phys. Rev. B **77**, 155127 (2008)
37. Stanić, I., Milovanović, M.V.: Phys. Rev. B **71**, 035329 (2005)
38. Sundermeyer, K.: Constrained Dynamics. Springer, Berlin (1982)
39. 't Hooft, G., Velman, M.: Diagramar. CERN (1973)
40. Tőke, C., Jain, J.K.: Phys. Rev. B **75**, 245440 (2007)
41. Tőke, C., Jain, J.K.: Phys. Rev. B **76**, 081403(R) (2007)
42. Tőke, C., Regnault, N., Jain, J.K.: Phys. Rev. Lett. **98**, 036806 (2007)
43. Tőke, C., Shi, C., Jain, J.K.: Phys. Rev. B **77**, 245305 (2008)
44. Tyutin, I.V.: Lebedev preprint FIAN 39, unpublished (1975) (in Russian)
45. Wilczek, F.: Phys. Rev. Lett. **49**, 957 (1982)
46. Ye, J.: Ann. Phys. **323**, 580 (2008)
47. Zhang, S.-C., Hansson, T.H., Kivelson, S.A.: Phys. Rev. Lett. **62**, 82 (1989)